*International Journal of Mathematics, Statistics and Operations Research*
Volume 2; Number 1; 2022 : pp. 71-78
*https://DOI: 10.47509 /IJMSOR.2022.v02i01.06*

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*www.arfjournals.com*

# INSURANCE CLAIM, RENEWAL AND SURPLUS PROCESSES IN DEPENDENT RISK MODEL THROUGH RUIN THEORY

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ARTICLEINFO

Received: 28 April 2022
Revised: 30 May 2022
Accepted: 11 June 2022
Online: 23 June 2022

*To cite this paper:*
G. Jagath Prasad Sreedhar & K.K. Suresh (2022). Insurance Claim, Renewal and Surplus Processes in Dependent Risk Model through Ruin Theory. *International Journal of Mathematics, Statistics and Operations Research.* 2(1): pp. 71-78. https://DOI: 10.47509 / IJMSOR.2022.v02i01.06

ABSTRACT

The main trust of this research paper is consideration of Insurance claim, renewal and surplus processes in dependent risk model through Ruin theory. Insurance companies maintain prosperity through careful design of premium rates. The premium rates are primarily based on the claims history adjusted to evolving various factors, such as the number of customers and their policies the returns from the investments in the financial market. This allows us to recover the Ruin probabilities theory obtained for general premiums dependent on reserve customers. Compare them with the asymptotic of the equivalent Ruin probabilities when the premium rate is fixed over time, to measure the expansion generated by this additional mechanism of binding the premium rates with the amount of reserve owned by the insurance company.

## INTRODUCTION

The insurance industry exists because people are willing to pay a price for being insured. There is an economic theory that explains why insured are willing to pay a premium larger than the net premium, that is, the mathematical expectation of the insured loss. This theory postulates that a decision maker, generally without being aware of it, attaches a value u(w) to his wealth w instead of just w, where u(·) is called his utility function

Insurance companies maintain solvency via careful design of premium rates. The premium rates are primarily based on the claims history and carefully adjusted to evolving factors, such as the number of customers and/or the returns from the investments in the financial market. Collective risk models, introduced by Lundberg and Cramér, describe the evolution of the surplus of an insurance business when considering constant premium rates, for the simplicity of arguments. This model, a compound Poisson process with drift, is referred to in the actuarial mathematics literature as the Cramér-Lundberg model. In practical situations, risk models with surplus-dependent premiums capture the dynamics of the surplus of an insurance company better. The Reference Lin and Pavlova (2006) advised for a lower premium for higher surplus levels to improve competitiveness, whereas a higher premium is needed for lower surplus levels to reduce the probability of ruin.

Among surplus-dependent premiums, risk models with *risky* investments have been widely analyzed (see e.g., Albrecher *et al.* 2012; Frolova *et al.* 2002; Paulsen 1993; Paulsen and Gjessing 1997). See Paulsen (1998) and Paulsen (2008) for surveys on the topic. The special case of risk models with *linearly* dependent premiums can be interpreted as models with *riskless* investments, since the volatility of return on investments, or the proportion of the capital invested in the risky asset is zero.

Under this scenario, exact expressions of the ruin probability are derived for compound Poisson risk models with interest on surplus and exponential-type upper bounds for renewal risk models with interest (see Cai and Dickson 2002, 2003). The Reference Cheung and Landriault (2012) investigated risk models with surplus-dependent premiums with dividend strategies and interest earning as a special case. Nirmala and Suresh (2018) proposed Designing of MATLAB Program for Various Fuzzy Quality Regions in CSP-MLP-T-3 Sampling plan. The Reference Czarna *et al.* (2019) discussed the ruin probabilities with the scale function from the theory of the Lévy process for risk models when the claim arrival process is a spectrally negative Lévy process and the premium rate function is non-decreasing and locally Lipschitz-continuous.

The traditional approach which is used to solve the ruin probability in risk theory is to establish the risk business model of insurance company. Asmussen summarized the main approaches and numerical results for the ruin probability with respect to many kinds of risk models. Embrechts and Schmidli discussed the ruin probability when the surplus process is a piecewise-deterministic Markov process. Cardosoa and Waters presented the numerical calculations of finite time ruin probabilities for extensions of the classical risk model. Gerber and Yang considered the compound Poisson insurance risk model perturbed by diffusion with investment. They proved that the absolute ruin probability satisfies a certain integro-differential equation, and some closed form solutions are obtained. Numerical methods of computing the ruin probability play an increasing role in actuarial sciences. Many researchers are seeking for more efficient numerical algorithm to study the issue by using new techniques and theories. Asmussen and Shakenov proposed the Monte Carlo algorithm to obtain the approximate solutions of the ruin probabilities in some risk models. Coulibaly discussed a simple quasi-Monte Carlo method to calculate the approximate solution of the ruin probability in the classical compound Poisson risk model.

Throughout this paper, we build on the method developed in Albrecher *et al.* (2013) to extend the derivation of ruin probabilities to surplus-dependent premium risk models with *Erlang* distributions (claim sizes or interarrival times). Recall from Albrecher *et al.* (2013), the risk model with surplus-dependent premiums described by

$$U(t) = u + \int_0^t p(u(s))ds - \sum_{k=1}^{N(t)} X_{kt}$$

where U(t) denotes the surplus at time t and p(U(t)) describes the premium rate at time t, a positive function of the current surplus U(t). When p(.) is constant, this model reduces to the classical collective risk model, see Asmussen and Albrecher (2010). As in the classical collective risk theory, ruin defines the first time the surplus becomes negative. For Tu, the time of ruin, given by

$$T_u = \inf \{t \geq 0 | u(t) \lhd 0|\}$$

The probability of ruin with initial value u is defined as

$$\varphi(u) = P\{T_u \leq \infty | u(0) = u|\}$$

Although it is impossible to determine a person's utility function exactly, we can give some plausible properties of it. For example, more wealth would imply a larger utility level, so u(0) should be a non-decreasing function. It is also logical that 'reasonable' decision makers are risk averse, which means that they prefer a fixed loss over a random loss with the same expected value. Define some classes of utility functions that possess these properties and study their advantages and disadvantages.

Ruin probability is one of the most important parts of insurance Statistics, which is not only the regulation of insurance companies to formulate strategies, but also the theoretical basis of preventing and reducing liquidation. Because it is of great practical significance and value to study the ruin probability and the related calculation problems, a considerable size of research work has been carried out about this problem.

## The Ruin probability Model

The ruin model describes the stability of an insurer. Starting from capital u at time t = 0, his capital is assumed to increase linearly in time by fixed annual premiums, but it decreases with a jump whenever a claim occurs. Ruin occurs when the capital is negative at some point in time. The probability that this ever happens, under the assumption that the annual premium as well as the claim generating process remain unchanged, is a good indication of whether the insurer's assets match his liabilities sufficiently. If not, one may take out more reinsurance, raise the premiums or increase the initial capital.

Analytical methods to compute ruin probabilities exist only for claims distributions that are mixtures and combinations of exponential distributions. Algorithms exist for discrete distributions with not too many mass points. Also, tight upper and lower bounds can be derived. Instead of looking at the ruin probability ø(u) with initial capital u, often one just considers an upper bound e"Ru for it (Lundberg), where the number R is the so-called adjustment coefficient and depends on the claim size distribution and the safety loading contained in the premium.

Computing a ruin probability assumes the portfolio to be unchanged eternally. Moreover, it considers just the insurance risk, not the financial risk. Therefore not much weight should be attached to its precise value beyond, say, the first relevant decimal. Though some claim that survival probabilities are 'the goal of risk theory', many actuarial practitioners are of the opinion that ruin theory, however topical still in academic circles, is of no significance to them. Nonetheless, we recommend to study at least the first three sections of Chapter 4, which contain the description of the Poisson process as well as some key results. A simple proof is provided for Lundberg's exponential upper bound, as well as a derivation of the ruin probability in case of exponential claim sizes.

## Ordering of risks

It is the very essence of the actuary's profession to be able to express preferences between random future gains or losses. Therefore, stochastic ordering is a vital part of his education and of his toolbox. Sometimes it happens that for two losses X and Y, it is known that every sensible decision maker prefers losing X, because Y is in a sense 'larger' than X. It may also happen that only the smaller group of all risk averse decision makers agree about which risk to prefer. In this case, risk Y may be larger than X, or merely more 'spread', which also makes a risk less attractive. When we interpret 'more spread' as having thicker tails of the cumulative distribution function, we get a method of ordering risks that has many appealing properties. For example, the preferred loss also outperforms the other one as regards zero utility premiums, ruin probabilities, and stop-loss premiums for compound distributions with these risks as individual terms.

Suppose that an insured can choose between an insurance policy with a fixed deductible and another policy with the same expected payment by the insurer and with the same premium. It can be shown that it is better for the insured to choose the former policy. If a reinsurer is insuring the total claim amount of an insurer's portfolio of risks, insurance with a fixed maximal retained risk is called a stop loss reinsurance. From the theory of ordering of risks of reinsurance is optimal for risk averse decision makers. In this chapter we prove that a stop-loss reinsurance results in the smallest variance of the retained risk. We also discuss a situation where the insurer prefers a proportional reinsurance, with a reinsurance payment proportional to the claim amount.

### Premium principles and risk measures

Assuming that the risk is known, or at least some characteristics of it like mean and variance, a premium principle assigns to the risk a real number used as a financial compensation for the one who takes over this risk. Note that we study only risk premiums, disregarding surcharges for costs incurred by the insurance company. By the law of large numbers, to avoid eventual ruin the total premium should be at least equal to the expected total claims, but additionally, there has to be a loading in the premium to compensate the insurer for making available his risk carrying capacity. From this loading, the insurer has to build a reservoir to draw upon in adverse times, so as to avoid getting in ruin. We present a number of premium principles, together with the most important properties that characterize premium principles. The choice of a premium principle depends heavily on the importance attached to such properties. There is no premium principle that is uniformly best. Risk measures also attach a real number to some risky situation. Examples are premiums, infinite ruin probabilities, one-year probabilities of insolvency, the required capital to be able to pay all claims with a prescribed probability, the expected value of the shortfall of claims over available capital, and more.

So the linear premium under Poisson model explicit form of Ruin Probability is,

$$\psi(u) \approx \frac{\mu}{\lambda c^{\lambda/e}} Ce^{-\mu u}(c+\varepsilon u)^{\lambda/e-1}, as u \to \infty$$

Nevertheless, the calculation of the probability of ruin is one of the central problems in insurance claim. The classical ruin model assumes that insurance claims arrive according to a Poisson process. In this setting it is possible to determine the moment generating function with the probability $1-\psi(u)$.

## CONCLUSION

The probability of ruin enables one to compare portfolios, but cannot attach any absolute meaning to the probability of ruin, as it does not actually represent the probability that the insurer will go bankrupt in the near future. First of all, it might take centuries for ruin to actually happen. Second, obvious interventions in the process such as paying out dividends or raising the premium for risks with an unfavourable claims performance are ruled out in the definition of the probability of ruin. Furthermore, the effects of inflation and return on capital are supposed to cancel each other out exactly.

The ruin probability only accounts for the insurance risk, not for possible mismanagement. Finally, the state of ruin is merely a mathematical abstraction: with a capital of –1, the insurer is not broke in practice, and with a capital of +1, the insurer can hardly be called solvent. As a result, the exact value of a ruin probability is not of vital importance; a good approximation is just as useful. Results reveal that the method can obtain the desired accuracy with only a few number of training points. It has been noted that our proposed Legendre neural network algorithm could be a good tool to solve the ruin probability.

### *Acknowledgment*

The Authors like to acknowledge, Bharathiar University for providing necessary facilities for pursuing Ph.D. research programme Statistics.

### *Financial support and sponsorship*

Nil.

### *Declaration of competing interest*

All authors declare that they have no conflict of interests

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